

Definitions

Definition: The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through point P with slope $m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ provided the limit exists.

Definition: The **derivative of f** is $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ provided the limit exists.

Differentiation Rules:

Constant Multiple Rule: $[c \cdot f(x)]' = c \cdot f'(x)$

Sum or Difference Rule: $[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

Product Rule: $[f(x) \cdot g(x)]' = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

Quotient Rule: $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Chain Rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

Fundamental Derivatives

Let c be any constant, and a be any positive constant.

$$\frac{d}{dx}c = 0$$

$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}x^c = c \cdot x^{c-1} \text{ (Power Rule)}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \cdot \ln a$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

$$\frac{d}{dx}\sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \cdot \ln a}$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \int_c^x f(t) dt = f(x)$$